Optimal free models of many-body interacting theories

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Motivation

- Interacting particles give rise to many fascinating phases of matter which remain theoretically challenging. Free systems are in contrast much better understood.
- Interacting systems are often described in terms of free and integrable models – as in mean field theory and perturbation theory.
- We quantify the relevance of interactions based on the particular structure of entanglement found in free systems.
- We use this measure to identify *optimal* free descriptions of interacting systems.

Interaction Distance

We introduce the *interaction distance* between an interacting state ρ and manifold of free states $\mathcal{F}$

$$T_{\mathcal{F}}(\rho) = \min_{\sigma\in\mathcal{F}} T(\rho,\sigma), \tag{1}$$

where $T(\rho,\sigma) = \frac{1}{2}\operatorname{tr}\sqrt{(\rho-\sigma)^2}$ is the trace distance. It measures the distinguishability of ρ from ground states of free theories.

The manifold $\mathcal{F}$ contains all unitary orbits of Gaussian states, or equivalently Gaussian states in any basis of mode operators $\{c\}$. In particular, σ can be free in terms of quasiparticles with different exchange statistics than those of ρ.

We have freedom to consider general unitary orbits because each σ can be expressed as

$$\sigma = \exp\left\{z + \sum_j m_j c_j^\dagger c_j\right\} \tag{2}$$

with some $\{c\}$ bosonic or fermionic mode operators. The action of a general unitary operator U on σ simply effects a canonical transformation $c \mapsto UcU^\dagger$ leaving the system free even if it is not a Gaussian map.

It has been shown that the trace distance is minimised within a unitary orbit when σ and ρ are simultaneously diagonal and their eigenvalues are both in rank order. Only minimisation with respect to $\{\epsilon\}$ is needed in order to compute interaction distance which makes this procedure efficient.

Free entanglement spectra

The reduced density matrix ρ for a ground state $|\psi\rangle$ is $\rho = \operatorname{tr}_B|\psi\rangle\langle\psi|$ found by tracing out part of the system. The entanglement Hamiltonian $H_E = -\ln\rho$ has eigenvalues $\{\xi\}$, known as the *entanglement spectrum*.

The entanglement spectrum of a free state is generated combinatorially from the *single-body* entanglement spectrum $\{\epsilon\}$,

$$\xi_n = \left\{\xi_0 + \sum_{i=1}^{N} n_i\epsilon_i\right\}_{n_i=0,\ldots,m}, \tag{3}$$

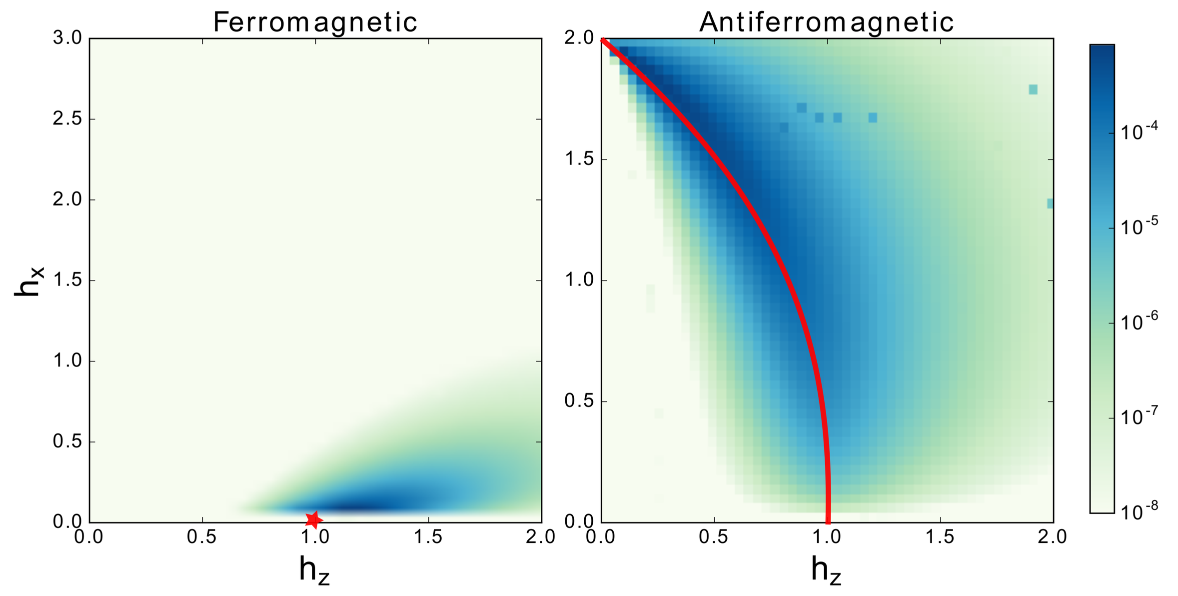
with $m=\infty$ for bosons, $m<\infty$ for soft-core bosons, and $m=1$ for fermions, and ξ_0 a normalisation constant. This is because the partial trace is a Gaussian map.

Case study: Ising model in a magnetic field

As an example we use the 1D quantum ferromagnetic (FM) and antiferromagnetic (AFM) Ising models in transverse (h_z) and (h_x) longitudinal fields

$$H_\pm = -\sum_j(\pm\sigma_j^x\sigma_{j+1}^x + h_z\sigma_j^z + h_x\sigma_j^x). \tag{4}$$

In both models the $h_x=0$ transvere Ising line is free and satisfies $T_{\mathcal{F}}=0$. This line contains a critical point at $h_x=0$, $h_z=1$ which is isolated in the FM model and connected by a critical line to the classical critical point $h_z=0$, $h_x=2$ in the AFM model (sketched).



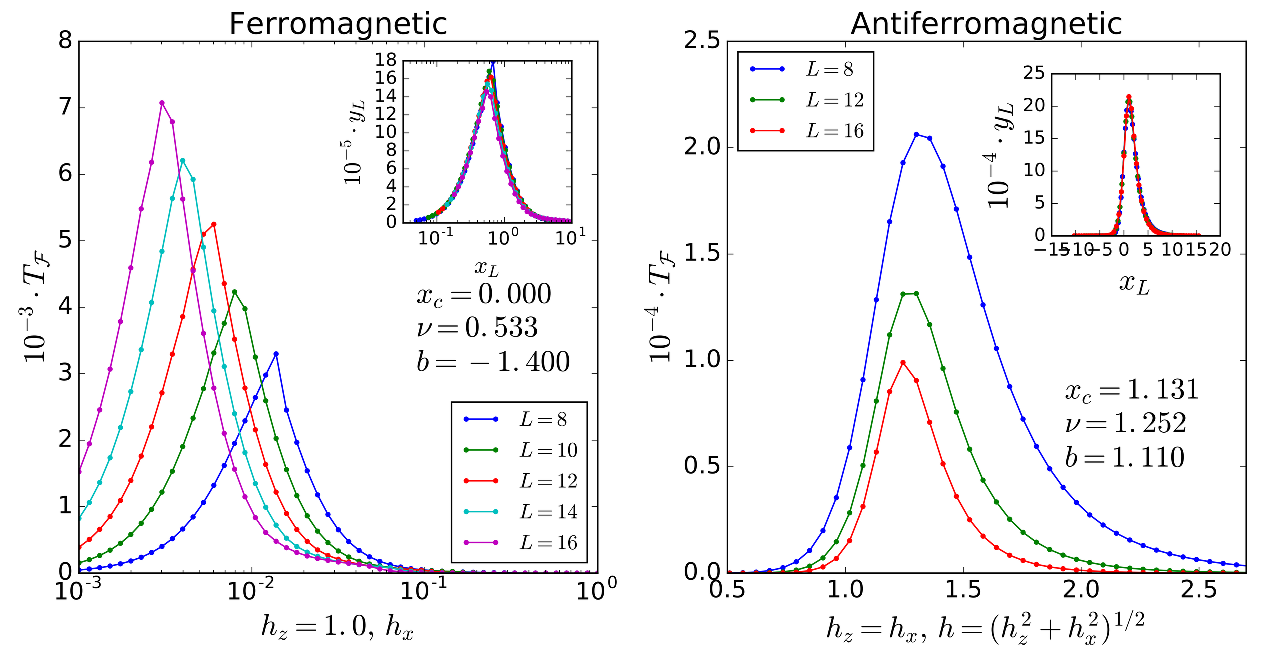
- The figures are for periodic boundary conditions and 16 spins.
- $T_{\mathcal{F}}$ is significant only close to criticality and decreasing toward zero approaching the stable fixed points of the renormalisation flow.
- As the system size is increased the region over which $T_{\mathcal{F}}$ is significant shrinks around criticality.

Scaling around critical points

We examine the scaling around criticality with finite size scaling (FSS) analysis according to

$$T_{\mathcal{F}}(\lambda;L) \sim L^{-b}\tilde{f}\left((h-h_c)L^{1/\nu}\right) \tag{5}$$

The exponent ν is the correlation length critical exponent. Sign of exponent b relates to the relevance or irrelevance of corresponding operator in the critical theory.



- From FSS analysis of the energy gap ΔE we find $\nu\approx 1.052$ for the AFM model which is within 20% of the value obtained from $T_{\mathcal{F}}$.
- We know analytically ν for the FM model where it is $\nu = 8/15$ which is in good agreement.

Convergence

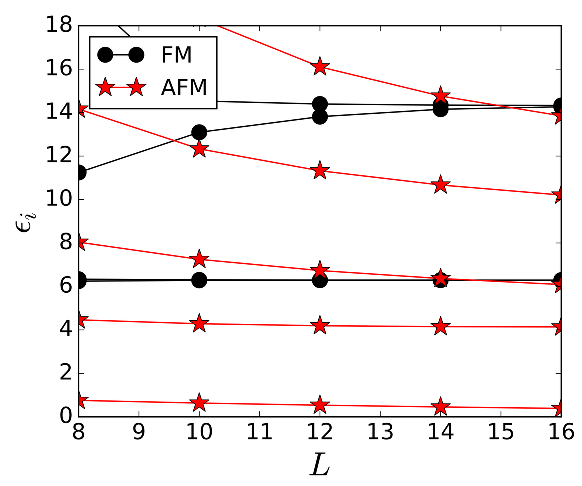
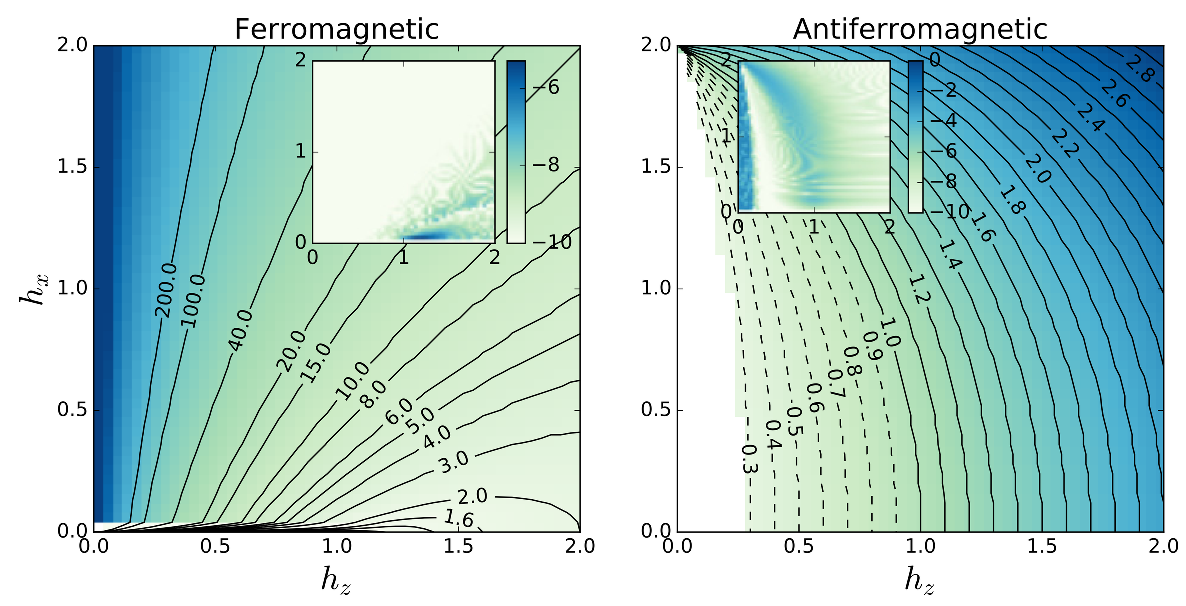


Figure shows the convergence the single particle entanglement energies ϵ as a function of system size L to their asymptotic values in the thermodynamic limit for the (blue) FM and (red) AFM models at $(h_x = 0.16, h_z = 0.88)$.

Matching the free model

We minimise the trace distance between the optimal free model and the free Ising models identifying the free theory $(h_z^{\mathcal{F}})$ with equivalent ground state correlations to the interacting theory (h_z, h_x).



- Contours shows matching of the optimal free model to the free Ising models in the thermodynamic limit.
- The Insets give the trace distance (log scale) between the optimal free model to the Ising free model.
- Unresolved degeneracy in the energy spectrum close to the classical axis $h_z=0$ causes artefacts which have been removed.

Outlook

- Could be used to find a free parent Hamiltonian.
- The scaling exponents around criticality could be explained using conformal field theory.
- See what happens for non-renormalisable models.

References

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