Weak ergodicity breaking from quantum many-body scars

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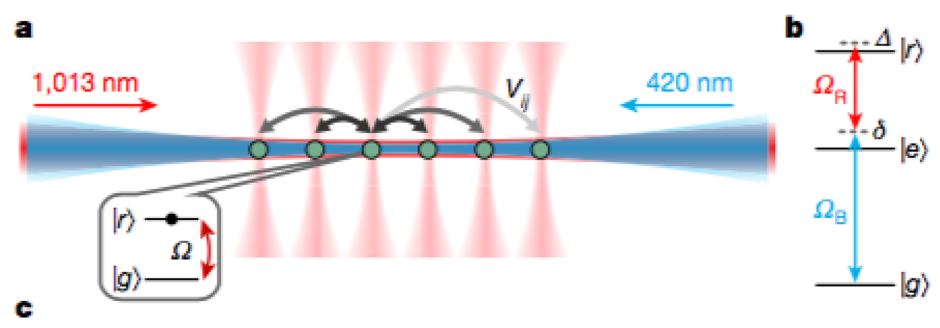
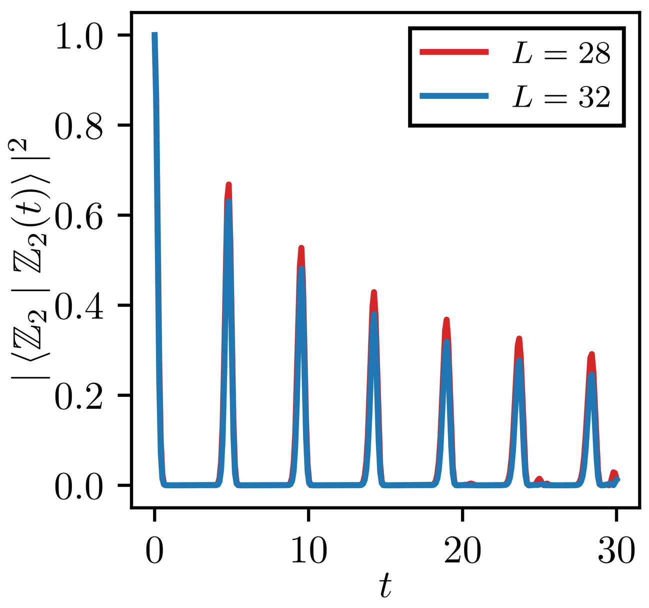
Experiment and motivation

A recent experiment [1] reports on a Rydberg chain with individual control over interactions. The Hamiltonian is

$$H = \sum_j \left(\frac{\Omega_j}{2} X_j - \Delta_j n_j\right) + \sum_{i<j} V_{ij} n_i n_j \qquad (1)$$

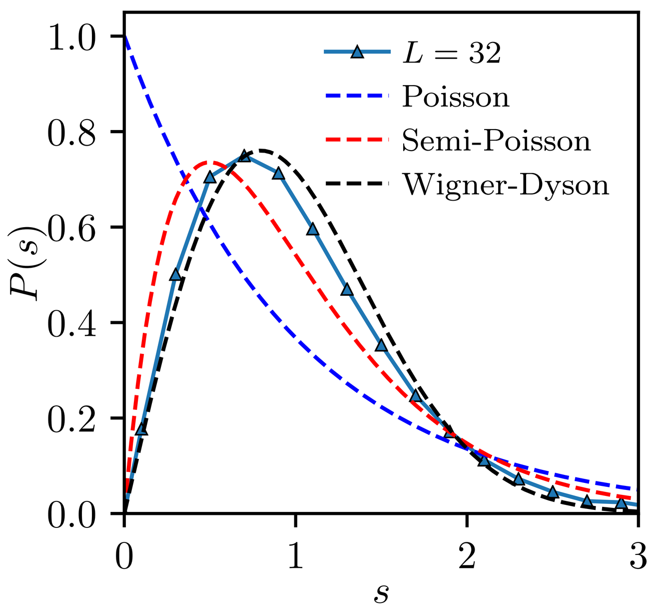
where couplings Ω is the Rabi frequency, Δ is a laser detuning and $V_{i,j} \sim C/r_{i,j}^6$ are replusive van der Waals interactions.



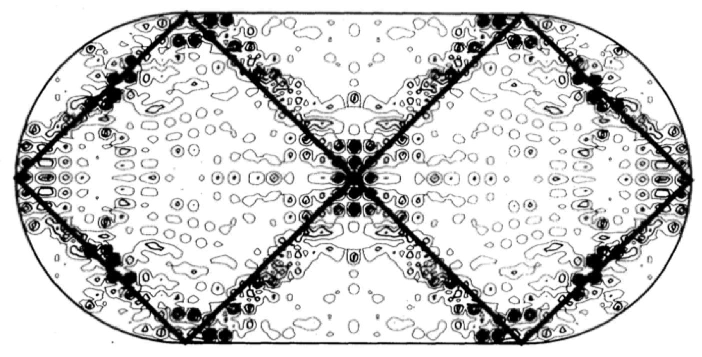


For homogeneous couplings and in the limit $V_{j,j+1} \gg \Omega \gg \Delta$ periodic quantum revivals were observed starting for a Néel initial state.

This is especially surprising considering that the system is non-integrable as evidenced by the level statistics! For large sizes ($L = 32$) it is approaches the Wigner-Dyson distribution.

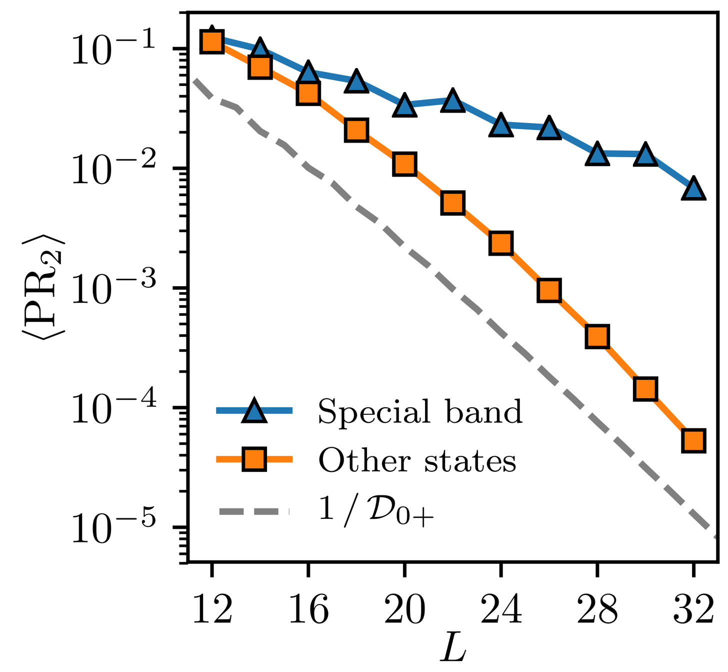


What is a quantum scar?



- Unstable periodic orbits of the classical stadium billiards (right) imprint upon a wavefunction (left) after quantisation[2].
- This is surprising! One might expect unstable classical period orbits to be lost in the transition to quantum mechanics as the particle becomes "blurred".
- Not all chaotic systems were created equivalent! This model is quantum ergodic but not quantum ***unique*** ergodic[3]. In the many-body setting think eigenstate thermalisation for all vs. ***almost*** all eigenstates.

Concentration in Hilbert space

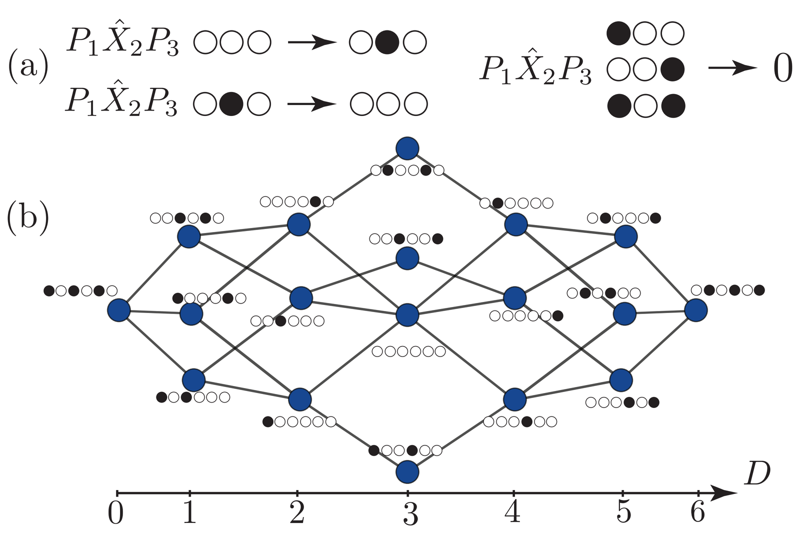


- This can be measured with the participation ratio

$$\mathrm{PR}_2 = \sum_\alpha |\langle \alpha \mid \psi\rangle|^4 \qquad (4)$$

in the product state basis.
- The special states are quite localised (they must have significant overlap with the Néel states).
- There are other states in each tower not in the band which are also somewhat localised and lifts the other states line from the delocalised prediction.

Effective model and constrained Hilbert space



In the same limit the Hamiltonian reduces to

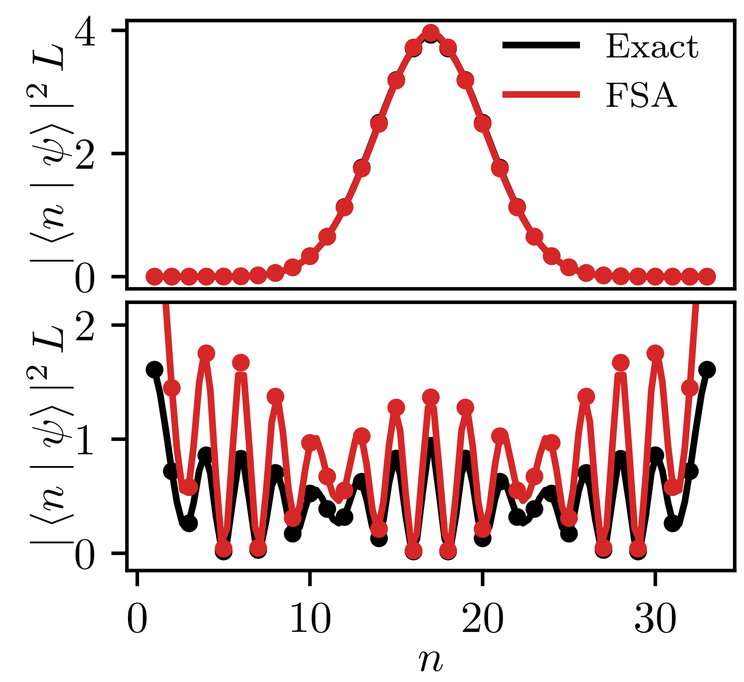
$$H = \sum_j P_j X_j P_j$$

where

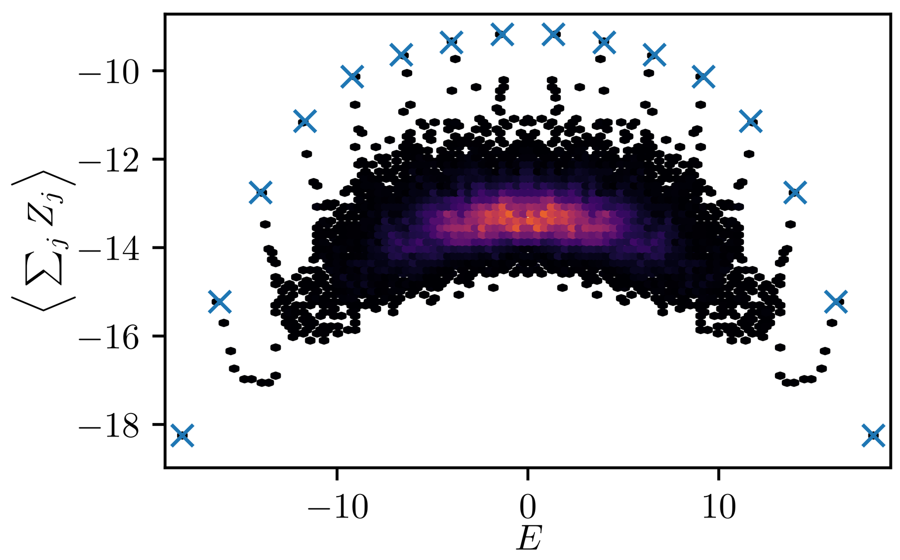
$$P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Adjacent excitations are forbidden. The Hilbert space is that of an effective model of Fibonacci anyons.

Scars and weak ergodicity breaking

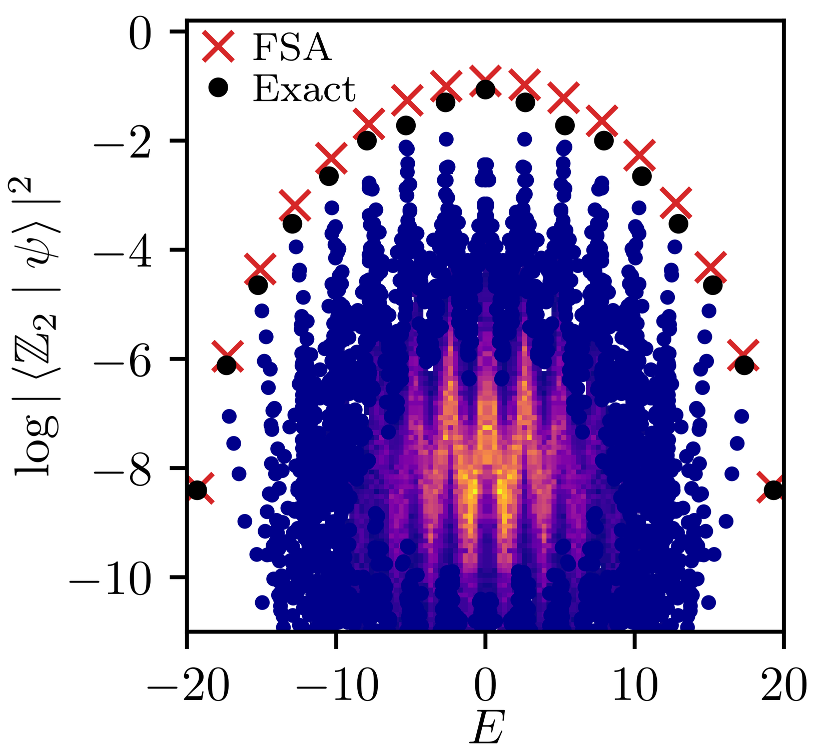


- The forward-scattering quasi-modes imprint upon the eigenstates forming a *many-body quantum scar*.
- Eigenstates in the special band are strongly scarred, those in the towers below are weakly scarred in the same way.
- The ground state is captured essentially exactly in the forward-scattering approximation.



- Matrix elements of local observables in the energy basis are not smooth w.r.t. energy.
- Strong formulations of the eigenstate thermalisation hypothesis are violated.

From dynamics to eigenstates



The eigenvalue decomposition reveals a band of special states which account for most of the Néel state with approximately equally spaced eigenvalues, and converging with system size. This explains the oscillatory dynamics.

Split the Hamiltonian $H = H_+ + H_-$ into a forward propagating part

$$H_+ = \sum_{j \text{ even}} X_j P_{j-1} Q_j P_{j+1} + \sum_{j \text{ odd}} X_j P_{j-1} P_j P_{j+1} \qquad (2)$$

and backward propagating part $H_- = H_+^\dagger$.

Build an orthonormal basis for the Krylov subspace generated by H_+ starting from the Néel state $\{|0\rangle, |1\rangle, \ldots, |L\rangle\}$. The Hamiltonian projected into this subspace is a tight-binding chain

$$H_{FSA} = \sum_{n=0}^{L} \beta_n \left(|n\rangle\langle n+1| + \text{h.c.}\right) \qquad (3)$$

with hopping amplitudes $\beta_n = \langle n+1| H_+ |n\rangle = \langle n| H_- |n+1\rangle$.

This is equivalent to a Lanczos recurrence with the approximation that the backward propagate is proportional to the previous vector $H_- |n+1\rangle \approx \beta_n |n\rangle$.

- Remarkably this can be carried out in time polynomial in L.
- Successfully identifies the most important states for explaining the oscillations.
- For $L = 32$ the eigenvalue error is $\approx 1\%$.
- The error in each step of the recurrence is $err(n) = |\langle n| H_+H_- |n\rangle/\beta_n^2 - 1|$ which for $L = 32$ has maximum $err(n) \approx 0.2\%$ and shows a decreasing trend with L.

Conclusions

To recap:-

- Non-integrable many-body system which displays periodic quantum revivals despite being ergodic.
- Approximate eigenvalues and eigenstate (quasi-modes) can be found which explain this effect.
- Further these quasi-modes scar the exact eigenstates signalling a failure of a strong eigenstate thermalisation hypothesis, i.e. ***almost all*** but not all the eigenstates are homogeneous, even in the middle of the band.

Also of interest:-

- We show number of zero energy states that grows with the Fibonacci numbers. Can be used for storing quantum information [4].
- Adding disorder leads to many-body localisation despite the non-tensor product Hilbert space structure [5].

References

[1] H. Bernien, et al. *Nature* **551** 7682, 579–584 (2017).
[2] E. J. Heller. *Physical Review Letters* **53** 16, 1515–1518 (1984).
[3] A. Hassell. *Annals of Mathematics* **171** 1, 605–618 (2010).
[4] M. Schecter, et al. arXiv:1801.03101.
[5] C. Chen, et al. arXiv:1709.04067.

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arXiv:1711.03528