APS March Meeting

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Operator-space fragmentation and integrability in Lindblad open quantum systems

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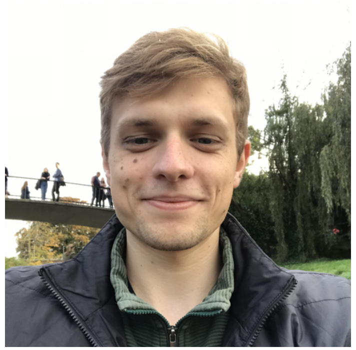


arXiv:2310.09406
arXiv:24xx.xxxxx

Motivation

- Interested in ergodicity breaking
  - Spontaneous symmetry breaking
  - Topological order and SPT
  - Many-body localisation
  - (Many-body) quantum scars
  - (Hilbert space) fragmentation
- What can we see in an open quantum system?
  - dissipation is fairly hostile to non-ergodic physics

Model

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -i\,[H,\rho] + \sum_j \kappa_j \left(2F_j\rho F_j^\dagger - \{F_j^\dagger F_j, \rho\}\right)$$

Hamiltonian terms $H = \sum_j Z_{j-1}X_jZ_{j+1}$ and jump operators $F_j = Z_{j-1}Z_{j+1}$.

These sums don't include any terms which would go over the open boundaries.

- Hamiltonian is an SPT phase.
- Zero modes are (approximate) strong symmetries before dissipation.
- Some of them then become *weak* symmetries after dissipation.
- Information is recoverable if you can make the jumps *observable*

This and more is all in arXiv:2310.09406 and not what this talk is about.

This isn't an SPT talk. It's a fragmentation talk.

Fragmentation

Bond $\mathcal{A}$ and commutant $\mathcal{C}$ algebras (see Moudgalya *et al.* PRX 2022)

$$\mathcal{A} = \langle u_j = -i\,\mathrm{ad}_{H_j},\, d_j = \mathrm{Ad}_{F_j} \rangle$$
$$\mathcal{C} = \{ O : [O, u_l] = 0, [O, d_l] = 0 \}$$

From the double commutant theorem we get a representation theoretic structure like the Schur-Weyl duality,

$$\mathcal{H} = \bigoplus_{\lambda} V_{\mathcal{A}}^{(\lambda)} \otimes V_{\mathcal{C}}^{(\lambda)}$$

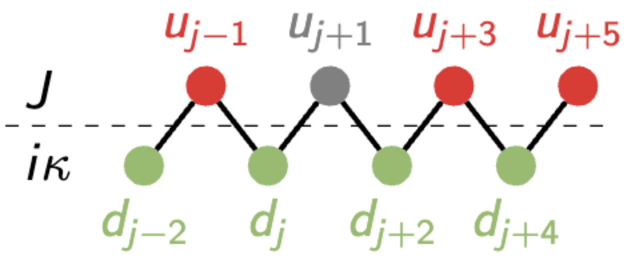
Operator dynamics of a Pauli M,

$$u_j M = -i\,\mathrm{ad}_{H_j} M = \begin{cases} 0 & \text{if } [H_j, M] = 0 \\ -2iH_j M & \text{otherwise} \end{cases} \qquad d_j M = \begin{cases} +M & \text{if } [F_j, M] \\ -M & \text{otherwise} \end{cases}$$

So $(u_j)^2 \in \mathcal{C}$

Fragmentation

The Lindblad terms either commute or anti-commute so we can summarise $\mathcal{A}$ represented onto a $(u_j)^2$ fragment with a frustration graph,

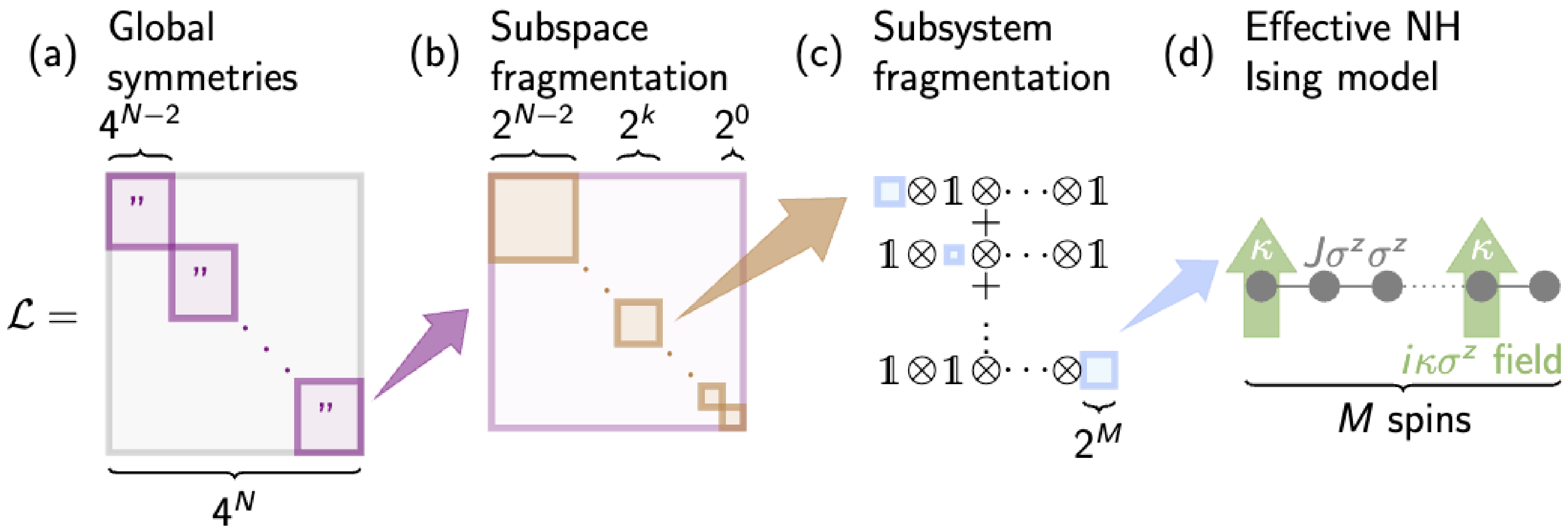


and choose another presentation for this algebra with the same properties (see Chapman *et al.* Quantum 2020). It's the TFIM!

- If $(u_j)^2 = 0$ in a representation then that Ising term is missing, dividing the system into subsystem fragments.
- Fields can be missing at the boundary due to open boundary conditions.
- There's another copy of the ising chain for the other parity of terms.

Fragmentation

In summary,



This is slightly coarser than that from the representation theory:

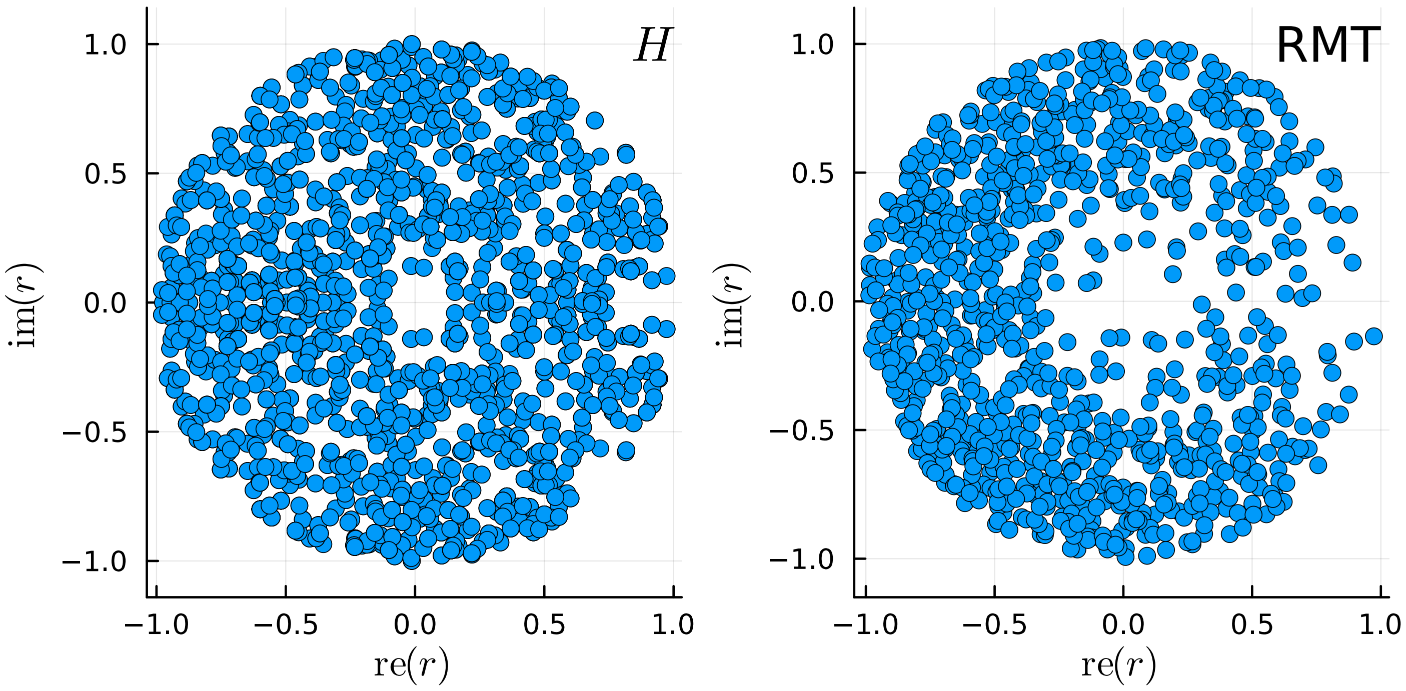
- $\mathcal{C}$ separates symmetry sectors of the Ising models.
- It also includes permutations among equivalent fragments.

Effective Model

- The effective model is a non-Hermitian transverse-field Ising model

$$H = \sum_j J\sigma_j^X \sigma_{j+1}^X + i\kappa \sum_j \sigma_j^Z$$

- Sometimes the boundary fields are missing again. This embeds the global zero modes from before.
- Complex level spacing ratio after resolving symmetries [Sá *et al.* PRX 2019]

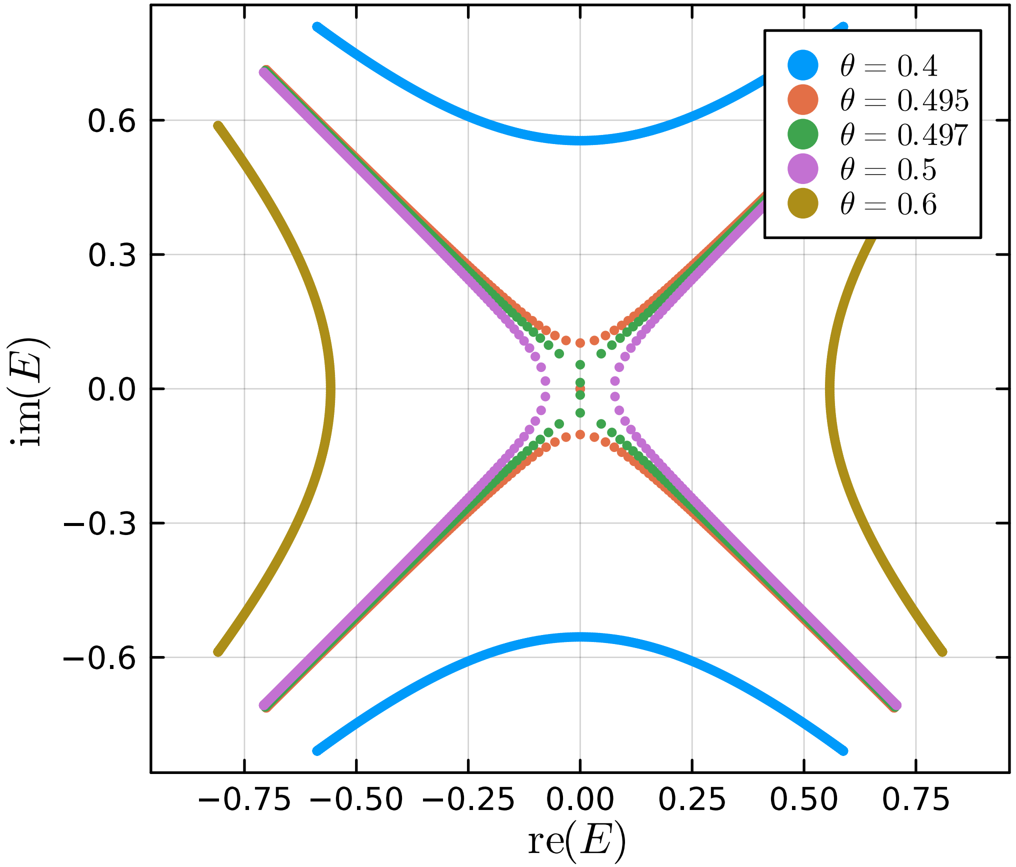


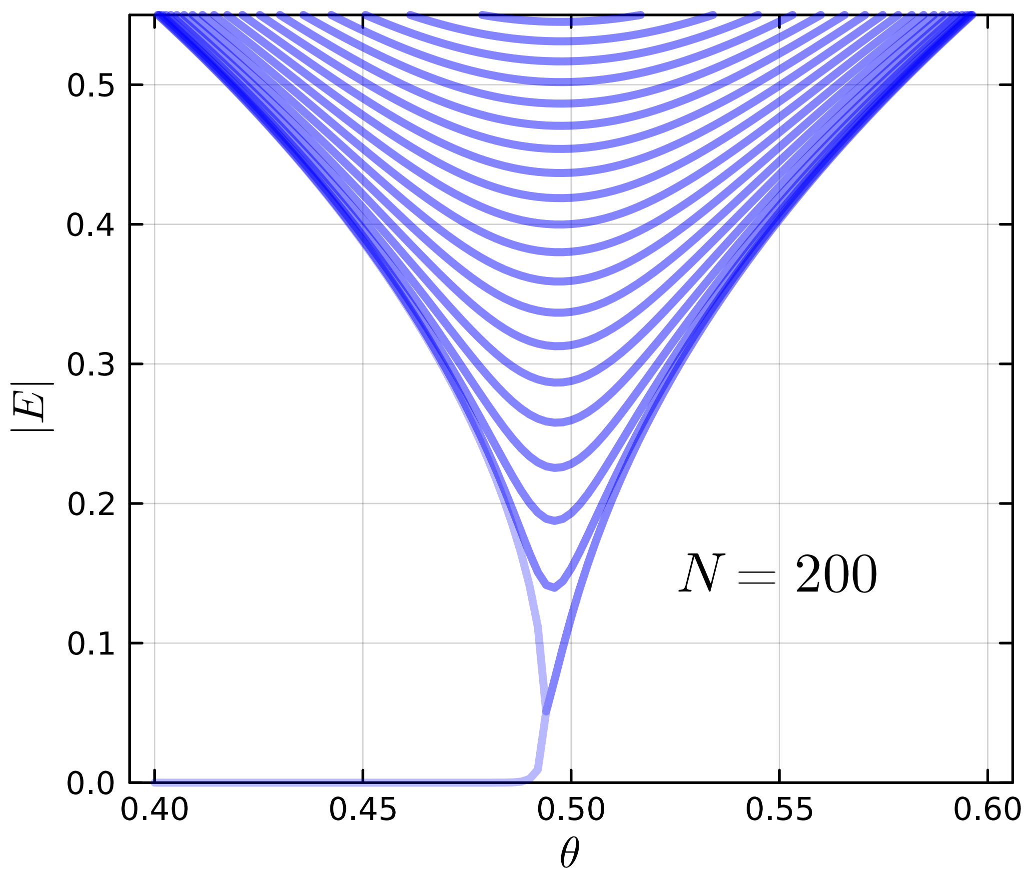
$$r_a = \frac{E_{\mathrm{nn}} - E_a}{E_{\mathrm{nnn}} - E_a}$$

Integrability and phase transition

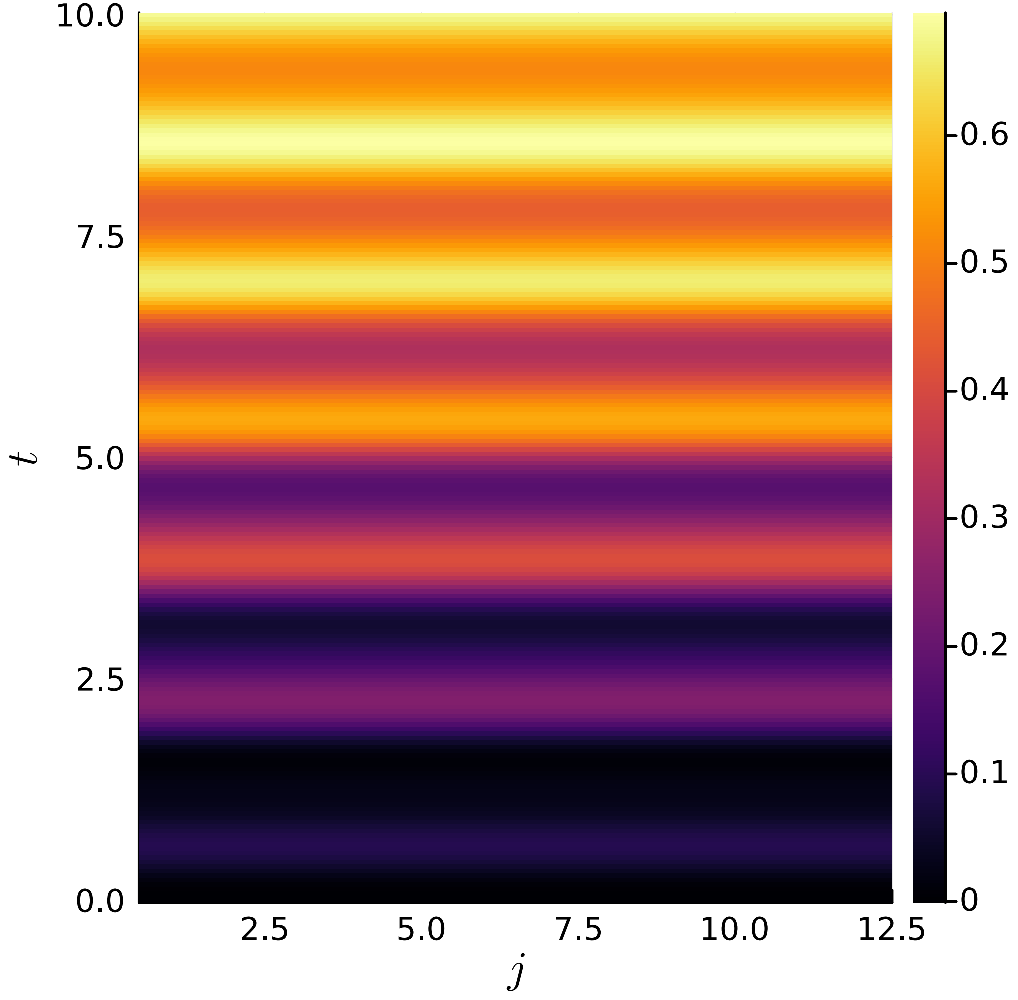
$$H = J\sum_j \gamma_{2j-1}\gamma_{2j} + i\kappa\sum_j \gamma_{2j}\gamma_{2j+1} = i\gamma^T A\gamma$$

$$J = \cos(\theta\pi/2),$$
$$K = \sin(\theta\pi/2)$$





Dynamical consequences



- Quench from (rapid cooling) ground state $|\phi\rangle$ of κ-dominated phase to J-dominated phase.
- $|\phi\rangle$ is roughly an extremal Y eigenvector.
- Looking at observables $\langle\phi(t)|\,\sigma_j^Z\,|\phi(t)\rangle$.
- Dynamic phase transition, oscillations in the κ order parameter.
- Eventually dissipation will win and system equillibrates.

Conclusions

To recap:-

- Fragmentation can naturally be generalised to Lindblad master equation as operator-space fragmentation. Perhaps "Hilbert-Schmidt space fragmentation".
- We can observe a non-Hermitian dynamical phase transition in the operator dynamics

Outlook:-

- What does this (or any) fragmentation mean for trajectories?
- Can we create an example with non-ergodic stationary states or interesting metastability?

See also: Essler *et al.* PRE 2020 which does something similar but with only dissipation and no unitary dynamics.