Quantum many-body scars

or *Non-ergodic Quantum Dynamics in Highly Excited States of a Kinematically Constrained Rydberg Chain*

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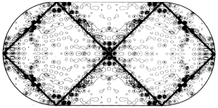
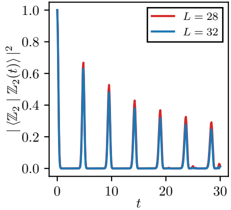
Outline

What is a quantum scar?

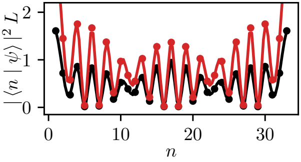


An experimental phenomena

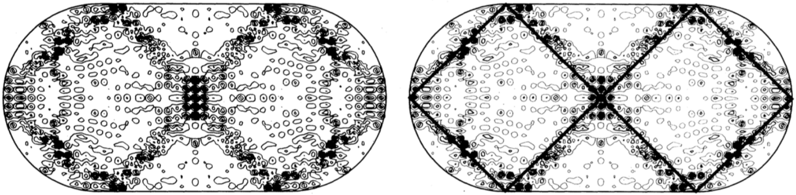


Why is it happening?

What else is going on?



Quantum scars



- First discussed by Heller 1984 in quantum stadium billiards.
- Here, classically unstable periodic orbits of the stadium billiards (right) scarring a wavefunction (left).
- One might expect unstable classical period orbits to be lost in the transition to quantum mechanics as the particle becomes "blurred".
- This model is quantum ergodic but not quantum *unique* ergodic[1]. Think eigenstate thermalisation for all eigenstates vs. *almost* all eigenstates.

[1] Hassell 2010.

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Probing many-body dynamics on a 51-atom quantum simulator

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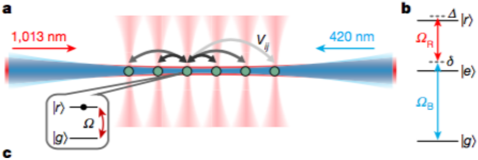
This experiment[2] reports on a Rydberg chain with individual control over interactions. The Hamiltonian is

$$H = \sum_j \left(\frac{\Omega_j}{2} X_j - \Delta_j n_j \right) + \sum_{i<j} V_{ij} n_i n_j \tag{1}$$

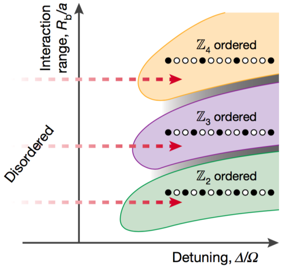
where couplings Ω is the Rabi frequency, Δ is a laser detuning and $V_{i,j} \sim C/r_{i,j}^6$ are replusive van der Waals interactions.

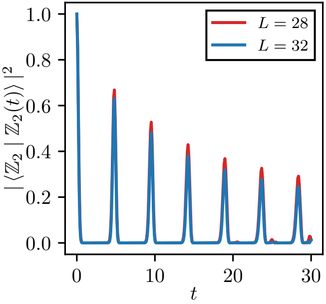


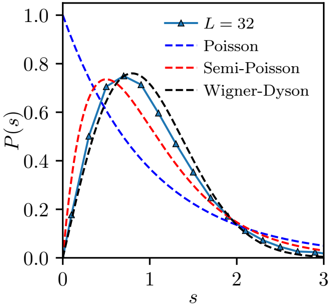
[2]See also another recent experiment Zhang et al. 2017 claiming 53 qubits

Quantum revivals

- For homogeneous couplings and in the limit $V_{j,j+1} \gg \Omega \gg \Delta$ periodic quantum revivals were observed.
- This is especially surprising considering that the system is non-integrable as evidenced by the level statistics.







An effective model

In this same limit the dynamics is generated by an effective Hamiltonian

$$H = \sum_j P_{j-1} X_j P_{j+1} \tag{2}$$

in an approximation well controlled up to times exponential in $V_{j,j+1}/\Omega$ which reproduces the same phenomena.
The Hilbert space of the model acquires a kinematic constraint. Each atom can be either in the ground $|\circ\rangle$ or the excited state $|\bullet\rangle$, but configurations where two adjacent atoms are both excited $|\cdots\bullet\bullet\cdots\rangle$ are forbidden. This makes the Hilbert space similar to that of chains of Fibonacci anyons[3].

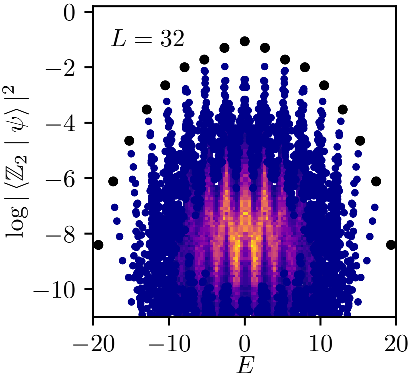
$$P_1\hat{X}_2P_3 \; \circ\circ\circ \to \circ\bullet\circ$$

$$P_1\hat{X}_2P_3 \; \circ\bullet\circ \to \circ\circ\circ$$

$$P_1\hat{X}_2P_3 \; \begin{matrix}\bullet\circ\circ \\ \circ\circ\bullet \\ \bullet\circ\bullet\end{matrix} \to 0$$

[3]Feiguin et al. 2007; Lesanovsky and Katsura 2012.

From dynamics to eigenvalues



- A band of special states which account for most of the Néel state.
- These have approximately equally spaced eigenvalues, and converging with system size.
- Explains the oscillatory dynamics.

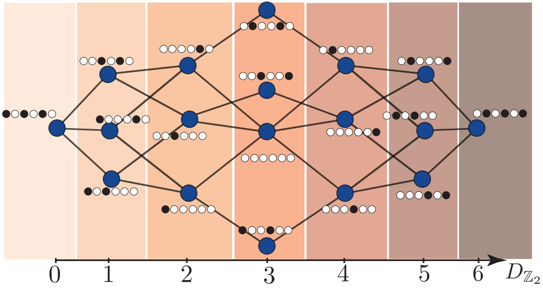
Goal: Find or otherwise explain these special states.

Forward-scattering approximation

Split the Hamiltonian $H = H_+ + H_-$ into a forward propagating part

$$H_+ = \sum_{j \text{ even}} X_j P_{j-1} Q_j P_{j+1} + \sum_{j \text{ odd}} X_j P_{j-1} P_j P_{j+1} \quad (3)$$

and backward propagating part $H_- = H_+^\dagger$. The forward-propagator increases distance from Néel state by one, and the backward-propagator decreases it.



Forward-scattering approximation

Build an orthonormal basis for the Krylov subspace generated by H_+ starting from the Néel state $\{|0\rangle, |1\rangle, \ldots, |L\rangle\}$. The Hamiltonian projected into this subspace is a tight-binding chain

$$H_{FSA} = \sum_{n=0}^{L} \beta_n \left(|n\rangle \langle n+1| + \text{h.c.} \right) \tag{4}$$

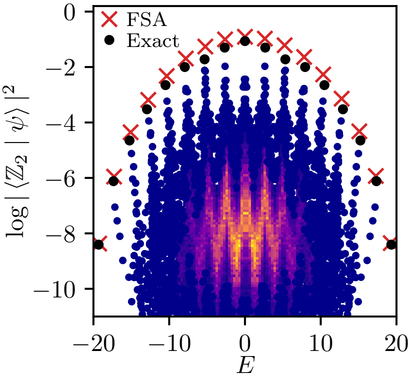
with hopping amplitudes

$$\beta_n = \langle n+1| H_+ |n\rangle = \langle n| H_- |n+1\rangle . \tag{5}$$

This is equivalent to a Lanczos recurrence with the approximation that the backward propagate is proportional to the previous vector

$$H_- |n+1\rangle \approx \beta_n |n\rangle . \tag{6}$$

Forward-scattering approximation



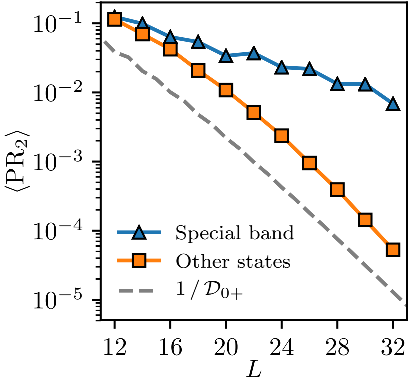
- Successfully identifies the important states for explaining the oscillations.
- For $L = 32$ the eigenvalue error $\Delta E/E \approx 1\%$.
- We can calculate eigenvalues and overlaps in this approximation scheme in time polynomial in L.

The error in each step of the recurrence is

$$err(n) = |\langle n| H_+ H_- |n\rangle / \beta_n^2 - 1| \quad (7)$$

which for $L = 32$ has maximum $err(n) \approx 0.2\%$ and a decreasing trend with N.

What else is going on? Concentration in Hilbert space



- This can be measured with the participation ratio

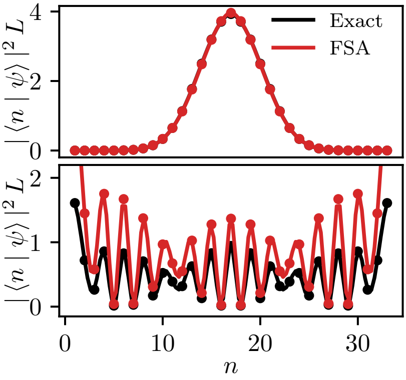
$$\mathrm{PR}_2 = \sum_{\alpha} |\langle \alpha \mid \psi \rangle|^4 \quad (8)$$

  in the product state basis.
- The special states are quite localised (they must have significant overlap with the Néel states).

- There are other states in each tower not in the band which are also somewhat localised and lifts the other states line from the delocalised prediction.

Quantum many-body scars

But what's *scarring* got to do with it?



- The forward-scattering quasi-modes imprint upon the eigenstates forming a *many-body quantum scar*.
- Eigenstates in the special band are strongly scarred, those in the towers below are weakly scarred in the same way.
- The ground state is captured essentially exactly in the forward-scattering approximation.

Conclusions

To recap:-

- Non-integrable many-body system which displays periodic quantum revivals despite being ergodic.
- Approximate eigenvalues and eigenstate (quasi-modes) can be found which explain this effect.
- Further these quasi-modes scar the exact eigenstates signalling a failure of a strong eigenstate thermalisation hypothesis, i.e. *almost all* but not all the eigenstates are homogeneous, even in the middle of the band.

Also of interest:-

- Number of zero energy states that grows with the Fibonacci numbers.